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Common Fixed Point Theorem under Complex Valued Metric Space

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Abstract—*In this paper , I prove the common fixed point theorem for a pair of mappings Satisfying rational type contractive conditions in frame work of complex valued metric Space. The prove results generalize and extended some of the know result in the literature.*

Keywords:- *contractive type mapping , complex valued metric space , common fixed point Rational contraction*

AMS Classification:- *54H 25, 47 H 10*

1. INTRODUCTION

One of the main area in the study of fixed point is metric fixed point theory, where the major and classical result was given prove by Banach [1], known as the Banach contraction principle, states that if (X,d) is a compete metric space and $T: X \to X$ is a contraction mapping i.e.

$d(x, y) \leq \alpha d(x, y)$ for all $x, y \in X$, where α is non negative number s.t. $\alpha < 1$. then T has a unique fixed point.

In 2011, Azam, A & fisher, B & Khan M. [2] introduced the complex valued metric space & Verma & Pathak [3]:,solanki et.al.[5], sintunavarat, cho., Kumam [4]: Chandok, S. [6,7,10]: Jungek, LG [11]; Sessa S [8]; Wintunavarat W [13]; Fouz kard F [12]; Nashin Inded Hashn [9] and many others. In this paper, we prove some common fixed point theorems for two pair of weakly mapping satisfy a contractive condition of rational type.

Theorem 1.1 let T be a continuous self map defined on a complete metric space (X, d). Suppose that T satisfies the following contractive condition.

$d(Tx, Ty) \leq \alpha \frac{d(y,Ty), d\,(x,Tx)}{d(x,y)} + \beta d(x, y), \quad \forall\, x, y \in X, x \neq y, \dots \dots 1.1.a$ where $, \beta \in [0, 1),\ s.t.\ \alpha + \beta < 1$. Then T has a unique fixed point

Also, in 1975 Dass & Gupta prove that every continues self map on the metric space (X, d) which satisfies the

$d(Tx, Ty) \leq \alpha \frac{d(y,Ty)[1+d\,(x,Tx)]}{1+d(x,y)} + \beta d(x, y), \quad \forall\, x, y \in X, \dots \dots 1.1.b$ where

$\alpha, \beta \in [0, 1),\ s.t.\ \alpha + \beta < 1$. Then T has a unique fixed point

2. PRELIMINARIES

Definition 2.1 [2] let c be the set of complex number and let $z_1, z_2, \in C$ as follows:

$$Z_1 \leq Z_2 \Leftrightarrow Re\,(Z_1) \leq Re(Z_2), Im\,(Z_1) \leq Im(Z_2) \dots .2.1.a$$

Consequently $Z_1 \leq Z_2$ if one of the following condition is satisfied

a) $Re\,(Z_1) = Re(Z_2), Im\,(Z_1) < Im(Z_2)$

b) $Re\,(Z_1) < Re(Z_2), Im\,(Z_1) = Im(Z_2)$

c) $Re\,(Z_1) < Re(Z_2), Im\,(Z_1) < Im(Z_2)$

d) $Re\,(Z_1) = Re(Z_2), Im\,(Z_1) = Im(Z_2)$

In particular $Z_1 \lneqq Z_2$ if $Z_1 \neq Z_2$ and one of (a), (b),(c) is satisfies and if $Z_1 < Z_2$

then only (c) is satisfied that

1. $a, b \in R \text{ and } a \leq b \Rightarrow aZ \leq bZ \ \forall Z \in C$
2. $0 \leq Z_1 \nleq Z_2 \Rightarrow |Z_1| < |Z_2|$
3. $Z_1 \leq Z_2 \text{ and } Z_2 < Z_3 \Rightarrow Z_1 < Z_3$

Definition 2.2 Let X be a non-empty set, & C be the set at complex numbers suppose that the mapping d: $X \times X \to C$ satisfies the following conditions

(i) $0 \leq d(x, y) \forall\, x, y \in X \,\&\, d\,(x, y) = 0 \ iff\ x = y$

(ii) $d(x, y) = d(y, x) \forall\, x, y \in X$

(iii) $d(x, y) \leq d(x, z) + d(z, y) \forall\, x, y \in X$

Then d is called a complex valued metric on X and (X, d) is called a complex valued metric space.

Definition 2.3 Let (X, d) be a complex valued metric space and let $\{x_n\}$ be a sequence in X. Then $\{x_n\}$ converge to x iff

$$|d(x_n, x)| \to 0 \quad \text{as } n \to \infty$$

Definition 2.4 Let (X, d) be a complex valued metric space and let $\{x_n\}$be a sequence in X. Then $\{x_n\}$ is a cauchy sequence iff

$$|d(x_n, x_{n+m})| \to 0 \quad \text{as } n \to \infty \text{ where } m \in N.$$

3. MAIN RESULT :

Theorem 3.1 Let (X, d) be a complete complex valued metric space and let the mapping $F, G : X \to X$ satisfies the condition.

$$d(Fx, Gy) \le \alpha d(x, y) + \beta \frac{d(x,Fx)d(x,Gy)+d(y,Gy)d(y,Fx)}{d(x,y)} + \gamma \frac{d(x,Fx)d(y,Gy)}{d(x,y)} + \delta[d(Gy, x) + (Fx, y)] \quad ....3.1.1$$

for all $x, y, \in X$ s. t. $x \ne y$, $d(x, y) \ne 0$ where $\alpha, \beta, \gamma, \delta$ are non negative reals with $\alpha + \beta + 2\gamma + 2\delta < 1$ or d(Fx, Gy) = 0 If $d(x, y) = 0$. Then F & G have a unique common fixed points.

Proof: Let x_0 be on a arbitrary point in X and define $x_{2k+1} = Fx_{2k}$;

$x_{2k+2} = Gx_{2k+1}$ *where* $k = 0, 1, 2,3 \dots ....$ Then

$$d(x_{2k+1}, x_{2k+2}) = d(Fx_{2k}, Gx_{2k+1})$$

$$\le \alpha d(x_{2k}, x_{2k+1}) + \beta \frac{d(x_{2k}, Fx_{2k})d(x_{2k}, Gx_{2k+1}) + d(x_{2k+1}, Gx_{2k+1})d(x_{2k+1}, Fx_{2k})}{d(x_{2k}, x_{2k+1})} + \gamma \frac{d(x_{2k}, Fx_{2k})d(x_{2k+1}, Gx_{2k+1})}{d(x_{2k}, x_{2k+1})} + \delta[d(Gx_{2k+1}, x_{2k}) + (Fx_{2k}, x_{2k+1})]$$

$$d(x_{2k+1}, x_{2k+2}) \le \alpha d(x_{2k}, x_{2k+1}) + \beta \frac{d(x_{2k},x_{2k+1})d(x_{2k},x_{2k+2})+d(x_{2k+1}x_{2k+2})d(x_{2k+1},x_{2k+1})}{d(x_{2k},x_{2k+1})} + \gamma \frac{d(x_{2k},x_{2k+1})d(x_{2k+1},x_{2k+2})}{d(x_{2k},x_{2k+1})} + \delta[d(x_{2k+2}, x_{2k}) + (x_{2k+1}, x_{2k+1})]$$

$$= \alpha d(x_{2k}, x_{2k+1}) + \beta d(x_{2k}, x_{2k+2}) + \gamma d(x_{2k+1}, x_{2k+2}) + \delta d(x_{2k+2}, x_{2k})$$

$$= (\alpha + \beta + \delta)d(x_{2k}, x_{2k+1}) + (\beta + \gamma + \delta)d(x_{2k+1}, x_{2k+2})$$

$$d(x_{2k+1}, x_{2k+2}) \le \frac{(\alpha+\beta+\delta)}{1-(\beta+\gamma+\delta)} d(x_{2k}, x_{2k+1})$$

So that

$$|d(x_{2k+1}, x_{2k+2})| \le \frac{(\alpha + \beta + \delta)}{1 - (\beta + \gamma + \delta)} |d(x_{2k}, x_{2k+1})|$$

As by triangle inequality

$$|d(x_{2k+1}, x_{2k+2})| \le |d(x_{2k+1}, x_{2k})| + |d(x_{2k}, x_{2k+2})|$$

similarly :

$$d(x_{2k+3}, x_{2k+2}) = d(Fx_{2k+2}, Gx_{2k+1})$$

$$\le \alpha d(x_{2k+2}, x_{2k+1}) + \beta \frac{d(x_{2k+2},Fx_{2k+2})d(x_{2k+2},Gx_{2k+1})+d(x_{2k+1},Gx_{2k+1})d(x_{2k+1},Fx_{2k+2})}{d(x_{2k+2},x_{2k+1})} + \gamma \frac{d(x_{2k+2},Fx_{2k+2})d(x_{2k+1},Gx_{2k+1})}{d(x_{2k+2},x_{2k+1})} + \delta[+d(Gx_{2k+1}, x_{2k+2}) + (Fx_{2k+2}, x_{2k+1})]$$

$$= \alpha d(x_{2k+2}, x_{2k+1}) + \beta \frac{d(x_{2k+2},x_{2k+3})d(x_{2k+2},x_{2k+2})+d(x_{2k+1},x_{2k+2})d(x_{2k+1},x_{2k+3})}{d(x_{2k+2},x_{2k+1})} + \gamma \frac{d(x_{2k+2},x_{2k+3})d(x_{2k+1},x_{2k+2})}{d(x_{2k+2},x_{2k+1})} + \delta[+d(x_{2k+2}, x_{2k+2}) + (x_{2k+2}, x_{2k+1})]$$

$$d(x_{2k+3}, x_{2k+2}) \le (\alpha + \beta + \delta)d(x_{2k+2}, x_{2k+1}) + (\beta + \gamma)d(x_{2k+2}, x_{2k+3})$$

$$d(x_{2k+3}, x_{2k+2}) \le \frac{(\alpha+\beta+\delta)}{1-(\beta+\gamma)} (d(x_{2k+2}, x_{2k+1})$$

so that $|d(x_{2k+3}, x_{2k+2})| \le \frac{(\alpha+\beta+\delta)}{1-(\beta+\gamma)} |(d(x_{2k+2}, x_{2k+1})$

As by triangle inequality

$$|d(x_{2k+2}, x_{2k+3})| \le |d(x_{2k+2}, x_{2k+1})| + |d(x_{2k+1}, x_{2k+3})|$$

so that $|d(x_{2k+3}, x_{2k+2})| \le s |(d(x_{2k+2}, x_{2k+1})$ where s = $\frac{(\alpha+\beta+\delta)}{1-(\beta+\gamma)} < 1$

$$|d(x_{n+1}, x_{n+2})| \le s|d(x_n, x_{n+1})| \le \cdots \le s^{n+1}|d(x_0, x_1)|$$ so that

for any m > n

As by triangle inequality

$|d(x_n,x_m)| \le$
$|d(x_n,x_{n+1})| + |d(x_{n+1},x_{n+2})| +$
$|d(x_{n+2},x_{n+3})| + \cdots + |d(x_{m-1},x_m)|$

$\le (s^n + s^{n+1} + \cdots + s^{m-1})\, |d(x_0,x_1)|$

$\le \frac{s^n}{1-s} |d(x_0,x_1)|$

hence. $|d(x_m,x_n)| \le \frac{s^n}{1-s} |d(x_0,Tx_0)|$
as m, n → ∞

This implies that $\{x_n\}$ is a cauchy sequence in X. Since X is complete, there exists some $v \in X$ such that $s_n \to v$ as $n \to \infty$.

suppose on the contrary that $v \ne Fv$,so that $d(v,Fv) = Z > 0$.

Now $d(v,Fv) = Z \le d(v,x_{2k+2}) + d(x_{2k+2},Fv)$

$\le d(v,x_{2k+2}) + d(Gx_{2k+1},Fv)$

$\le d(v,x_{2k+2}) + \alpha d(v,x_{2k+1}) + \beta \frac{d(v,Fv)d(v,Gx_{2k+1}) + d(x_{2k+1},Gx_{2k+1})d(x_{2k+1},Fv)}{d(v,x_{2k+1})} + \gamma \frac{d(v,Fv)d(x_{2k+1},Gx_{2k+1})}{d(v,x_{2k+1})} + \delta[d(Gx_{2k+1},v) + (Fv,x_{2k+1})]$

$= d(v,x_{2k+2}) + \alpha d(v,x_{2k+1}) + \beta \frac{Zd(v,x_{2k+2}) + d(x_{2k+1},x_{2k+2})d(x_{2k+1},Fv)}{d(v,x_{2k+1})} + \gamma \frac{Zd(x_{2k+1},x_{2k+2})}{d(v,x_{2k+1})} + \delta[d(x_{2k+2},v) + (Fv,x_{2k+1})]$

so that

$|d(v,Fv)| = |Z| \le |d(v,x_{2k+2})| + \alpha |d(v,x_{2k+1})| + \beta \frac{|Z|\,|d(v,x_{2k+2})| + |d(x_{2k+1},x_{2k+2})|\,|d(x_{2k+1},Fv)|}{|d(v,x_{2k+1})|} + \gamma \frac{|Z|\,|d(x_{2k+1},x_{2k+2})|}{|d(v,x_{2k+1})|} + \delta\left[|d(x_{2k+2},v)| + |d(Fv,x_{2k+1})|\right]$

which on mapping $n \to \infty$

Therefore $|d(v,Fv)| = 0$

which is contradiction so that $v = Fv$

similarly we show that $v = Gv$

Thus implies that v is fixed point

Uniqueness:

Let w in X be another common fixed point of F & G. Then

$$d(v,w) = d(Fv,Fw)$$
$$\le \alpha d(v,w) + \beta \frac{d(v,Fv)d(v,Gw) + d(w,Gw)d(w,Fv)}{d(v,w)} + \gamma \frac{d(v,Fv)d(w,Gw)}{d(v,w)} + \delta[d(Gw,v) + (Fv,w)]$$
$$= \alpha d(v,w) + \beta d(w,Gw) + \delta[d(Gw,v) + d(v,w)]$$
$$d(v,w) \le (\alpha + 2\delta) d(v,w)$$

$$|d(v,w)| \le (\rho) d(v,w)$$

where $\rho = \alpha + 2\delta < 1$ so $v = w$, which proves the uniqueness of common fixed point.

4. CONCLUSION

In this paper, we have established common fixed point result for Jaggi Type & Chatterjee Type contractive mapping in the context of complex valued metric space.

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